Chapter 2
Failure Modes (Behavior) and Wedge Sliding Analysis

2-1. General

The objective of a stability analysis is to maintain horizontal, vertical, and rotational equilibrium of the structure. Geologic information is needed to properly define and perform a realistic stability analysis. Possible failure modes and planes of weakness must be determined from onsite geological conditions, material strengths, and uplift forces. Stability is ensured by :

- Providing an adequate factor of safety against sliding at all possible failure planes.
- Providing specific limitations on the magnitude of the foundation bearing pressure.
- Providing restraints to the permissible location of the resultant force on any plane.
- Providing an adequate factor of safety against flotation of the structure.

However, satisfying the above provisions may not ensure stability if the structure experiences significant loss of foundation material due to erosion or piping, or if there is an internal failure due to inadequate strength of the concrete or steel materials. Stability is just one of the requirements necessary to ensure adequate structural performance.

2-2. Limit Equilibrium Analysis

The forces and pressures acting on a structure are highly indeterminate. Static equilibrium equations are insufficient to obtain a solution for lateral soil forces; additional assumptions must be incorporated in the analysis. For nonlinear materials, such as soils, this is commonly and conveniently done by assuming that a limit or failure state exists along some surface and that the shear force along the surface corresponds to the shear strength of the material. With these assumptions, equilibrium equations can be solved. Hence, this approach is commonly called limit-equilibrium analysis. To assure that the assumed failure does not occur, a reduction factor (safety factor or strength mobilization factor) is applied to the material strength. It should be noted that this approach differs significantly from that commonly used for indeterminate structure analysis, where stress-strain properties and deformations are employed. This limit equilibrium approach provides no direct information regarding deformations; it is implied that deformations are sufficient to induce the failure condition. Deformations are indirectly limited to tolerable values by the judicious selection of a safety factor.

2-3. Sliding Critical Planes

a. Contact surface. Sliding stability is based on the limit equilibrium method, which is an approximate nonlinear analysis method. Sliding safety must be assessed along selected surfaces within the structure. The selected surfaces for new designs would include along lift joints or any known weak planes while, for existing dams, additional surfaces will include any existing cracks. Sliding safety must also be assessed at/or near the foundation-structure interface. This surface may be either level or sloping. Generally, it may be assumed that a surface that slopes upward (in the direction of possible sliding) will have a beneficial effect, while one that slopes downward will increase the possibility for sliding. Figure 2-1 illustrates the beneficial and adverse effects of base slope.

b. Shallow weak planes. Where a shallow weak seam exists below a structure's contact with the foundation, two possible failure modes are present. One mode involves slippage along the weak plane (directly under the structure) and along its extension until it daylights. The other mode involves slippage along the weak plane directly under the

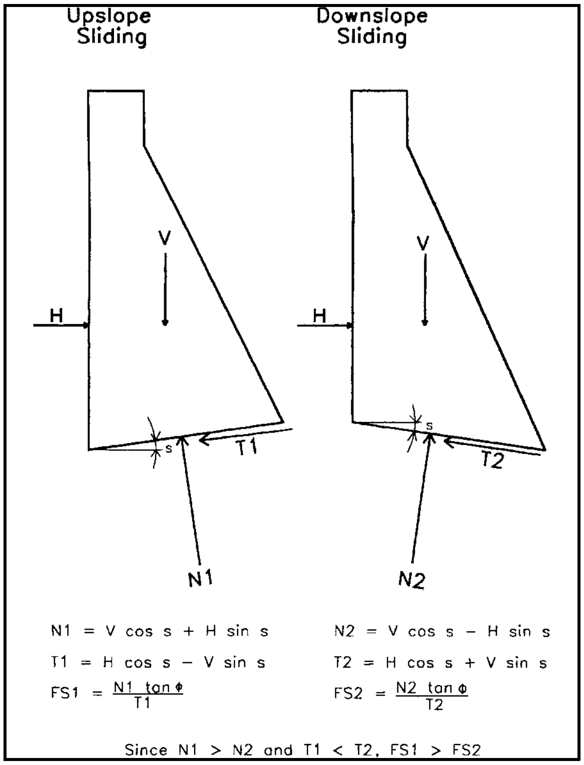


Figure 2-1. Sloping base planes

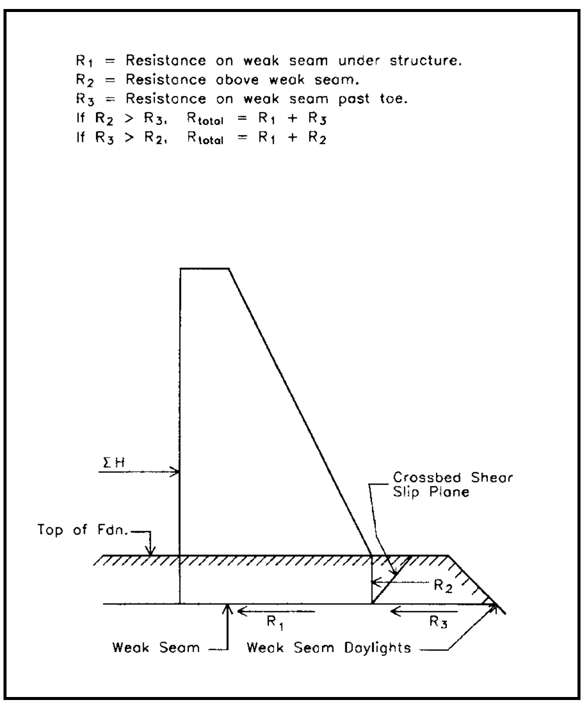


Figure 2-2. Weak plane with crossbed resistance

structure plus slip gauge along a plane through the foundation above the weak seam (crossbed shear for rock or passive resistance for soil). When the weak seam extends a large distance past the toe of the structure without daylighting, the second mode will usually be critical. Figure 2-2 illustrates these modes of failure.

c. Imbedded structure. The base of the structure may be imbedded below the top of the foundation when it is necessary to locate the structure on a stronger material. In this case, the second mode of failure illustrated in Figure 2-2, which involves slippage along the base plus slippage along a plane through the foundation at the toe of the structure (crossbed shear for rock or passive resistance for soil), will generally be critical.

2-4. Resultant Location

This guidance refers to *rotational stability* as the resultant location, and *conformance with resultant location requirements* implies the structure is safe from rotational failure. The slope of the resultant and its location are critical in assessing the foundation's bearing capacity. For some loading conditions, the resultant is allowed to fall outside the middle-third of the base. In these instances, it is assumed that the structure-foundation interface has no capability for resisting tensile stresses; therefore, part of the structure's base is assumed to lose contact with the foundation resulting in changes to the uplift pressure acting on the base.

2-5. Bearing

Analytical methods, traditional bearing capacity equations, and field load tests are all used to determine the bearing capacity of soil and rock. The allowable bearing capacity is defined as the maximum pressure that can be permitted on a foundation soil or rock mass giving consideration to all pertinent factors, with adequate safety against rupture of the soil or rock mass, or movement (settlement) of the foundation of such magnitude as to jeopardize the performance and safety of the structure. Increases in allowable bearing pressures are permitted for unusual and extreme load conditions over those required for usual load conditions. This is consistent with the approach established for structural performance. The allowable increase is discussed in Chapter 3.

a. Soil. For structures founded on soil, the bearing capacity is the ability of the soil to safely carry the pressure placed on the soil from the structure without undergoing a shear failure. Prevention of a shear failure, however, does not insure that settlements will be within acceptable limits; therefore, a settlement analysis is usually performed in addition to the bearing capacity analysis. Since settlement is not a part of stability, it is not included in this manual. Discussion on methods for estimating settlements and limitations in accuracy of settlement analyses is contained in EM 1110-1-1904. The bearing capacity of soils is covered in EM 1110-1-1905. General shear failure in a homogeneous soil foundation, for a vertical loading applied at the middle of the structure's base contact with the foundation, is illustrated by Prandtl's arc of shear failure as shown in Figure 2-3. Eccentricity of the load and horizontal components will affect the shape of the failure surface shown in the figure and tend to make this type of failure more probable.

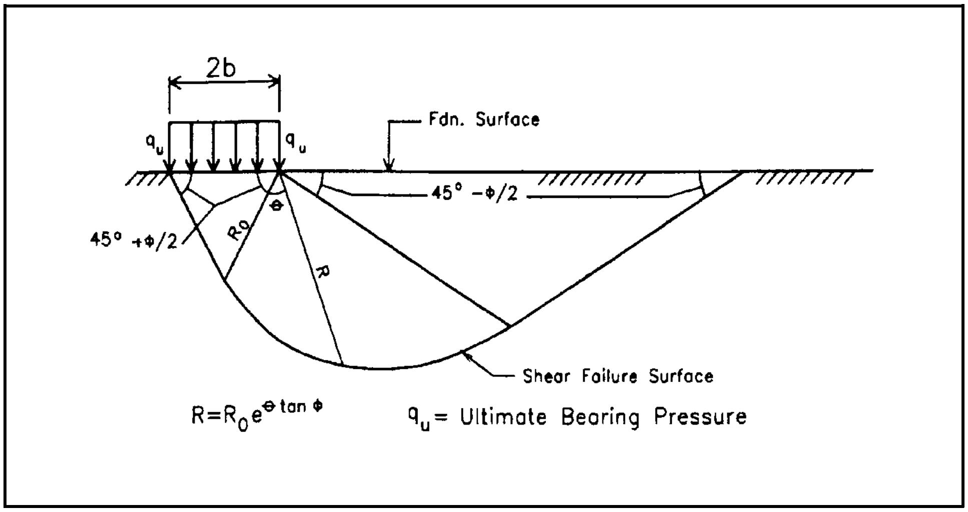


Figure 2-3. Prandtl s arc of failure

b. Rock. For structures founded on rock, failure modes may consist of local crushing, shear failures on weak seams, and failures at discontinuities or along bedding planes. The bearing capacity of the rock is often greater than the compressive strength of the concrete; therefore, the latter controls in the stability analysis. The bearing capacity of rock will depend on whether the rock is intact, jointed, layered, or fractured. The bearing capacity of rock foundations is covered in EM 1110-1-2908.

2-6. Flotation

This mode of failure occurs when the buoyant force (uplift) exceeds the summation of forces due to the weight of the structure, the weight of water contained in the structure, the weight of water above the top of the structure, the weight of soil that is part of the structural wedge, and any surcharge load.

2-7. Geotechnical Explorations and Testing

The scope of any geotechnical investigation will depend on geological structural complexity, imposed or existing loads acting on the foundation, and to some extent the consequences should a failure occur. Geotechnical explorations and foundation investigations may require many drill holes to accurately define the location, orientation, and composition of all faults and shear zones as well as providing drill cores for testing to establish the physical properties of intact and sheared foundation materials. The physical properties established through testing include density, modulus of elasticity, shear strength, bearing strength, and permeability. The complexity of the foundation will determine how many drill holes are required, mapping, trenching, and other exploratory measures must be undertaken to accurately describe foundation conditions. Guidelines for foundation explorations and testing are provided in EM 1110-1-1804, EM 1110-1-1802, and EM 1110-1-2908.

2-8. Shear Strength Parameters

Shear strength parameters, *phi* and *c*, are most often established by direct shear and triaxial testing of drill core specimens. Sometimes in situ testing is used to verify the results obtained through laboratory testing. Shear strength is a function of the degree of compaction for soils and a function of the confining pressure for rock. Therefore, any tests performed in the laboratory should model the conditions the foundation will experience during project operation. Since shearing may take place on a plane that includes intact rock, sheared rock, and jointed rock, strength values for all differing rock conditions must be established for use in determining a sliding factor of safety. Methods for determining shear strength values for rock foundations are provided in EM 1110-1-2908. Methods for determining shear strength values for soil foundations are provided in EM 1110-2-1906.

2-9. Strain Compatibility

The designer must be aware of the displacements required to reach the peak shear strengths of the various foundation and backfill materials, as well as the displacements that are associated with residual shear strength. With varied foundation conditions, it may not be possible to have all the foundation materials at their peak strengths at the same displacement (see Chapter 6 below). In those conditions, and for conditions that rely on passive resistance of a rock wedge or soil backfill, the engineer performing the stability analyses must make sure the strength values used are consistent with the displacements that will put the structure at the limit state assumed for the sliding stability analysis. The designer should be aware of all the limitations pertaining to the stability analysis procedure that is being used. A discussion of the sliding equilibrium method and its limitations can be found in EM 1110-1-2908.

2-10. Multiple-Wedge Sliding Analysis

The multiple-wedge sliding analysis is a fairly simple assessment of the sliding factor of safety along the various critical planes discussed in paragraph 2-3 above. It can account for the behavior expected from complex soil stratification and geometry. This method of analysis is illustrated in Appendix D, example D2.

a. Multiple-wedge design process.

(1) Analysis. An adequate assessment of sliding stability must account for the basic structural behavior, the mechanism of transmitting compressive and shearing loads to the foundation, the reaction of the foundation to such loads, and the secondary effects of the foundation behavior on the structure.

(2) Coordination. A fully coordinated team of geotechnical and structural engineers and geologists should insure that the result of the sliding analyses is properly integrated into the overall design of the substructure. Some of the critical aspects of the design process which require coordination are:

(a) Preliminary estimates of geotechnical data, subsurface conditions, and types of substructures.

(b) Selection of loading conditions, loading effects, potential failure mechanisms, and other related features of the analytical models.

(c) Evaluation of the technical and economic feasibility of alternative substructures.

(d) Refinement of the preliminary substructure configuration and proportions to consistently reflect the results of detailed geotechnical site explorations, laboratory testing, and numerical analyses.

(e) Modification of the substructure configuration or features during construction due to unexpected variations in the foundation conditions.

b. Method of analysis.

(1) Application of factor of safety. The guidance for the multiple-wedge sliding analysis is based on modern principles of structural and geotechnical mechanics that apply a safety factor to the material strength parameters in a manner which places the forces acting on the structure and foundation wedges in sliding equilibrium.

(2) Basic concepts and principles.

(a) A sliding mode of failure will occur along a presumed failure surface when the applied shearing force exceeds the resisting shearing forces. The failure surface can be any combination of plane and curved surfaces, but for simplicity, all failure surfaces are assumed to be planes which form the bases of wedges.

(b) The critical failure surface with the lowest safety factor can be determined by an iterative process. However, a single-step analysis, using the required minimum factor of safety, can be used as a simple pass/fail test.

(c) Sliding stability of most concrete structures can be adequately assessed by using a limit equilibrium approach. Designers must exercise sound judgement in performing these analyses.

(3) Assumptions and simplifications.

(a) A two-dimensional analysis is presented. These principles should be extended if unique three-dimensional geometric features and loads critically affect the sliding stability of a specific structure.

(b) Only force equilibrium is satisfied in this analysis, moment equilibrium is not ensured.

(c) The shearing force acting parallel to the interface of any two wedges depends on the slope angles at the top of the wedges. These shear forces are assumed to have a negligible effect; therefore, the failure surface at the bottom of each wedge is only loaded by the forces directly above it. (This assumption may not apply when considering vertical shears as illustrated in Appendix F.)

(d) Analyses are based on assumed-plane failure surfaces. The calculated safety factor will be realistic only if the assumed failure mechanism is kinematically possible.

(e) Considerations regarding displacements are excluded from the limit equilibrium approach. The relative rigidity of different foundation materials and the concrete substructure may influence the results of the sliding-stability analysis. Such complex structure-foundation systems may require a more intensive sliding investigation than a limit equilibrium approach. The effects of strain compatibility along the assumed failure surface may be included by interpreting data from in situ tests, laboratory tests, and finite element analyses.

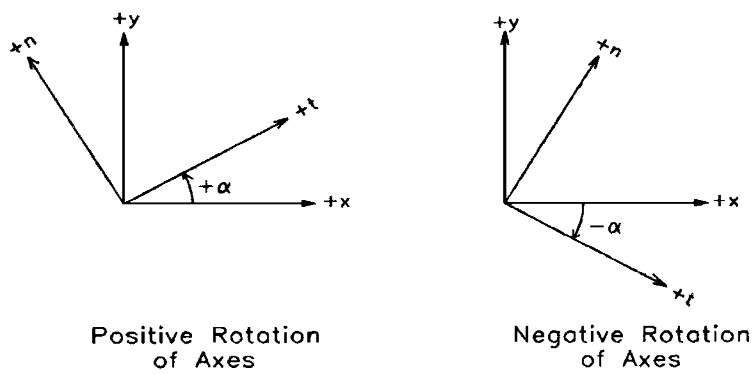
(f) A linear relationship is assumed between the resisting shearing force and the normal force acting on the failure surface beneath each wedge.

(g) The maximum shear strength that can be mobilized is adequately defined by the Mohr-Coulomb failure theory.

(h) The factor of safety is defined by Equation 2-7.

c. Analytical techniques for multi-wedge systems.

(1) Derivation of governing wedge equation. Derivation of the governing wedge equation for a typical wedge is shown in paragraph 2-10*f*. The sign convention used for the geometry in the derivation is shown in Figure 2-4. A general procedure for analyzing multi-wedge systems includes:



The equations for sliding stability analysis of a general wedge system are based on the right hand sign convention which is commonly used in engineering mechanics. The origin of the coordinate system for each wedge is located in the lower left hand corner of the wedge. The x and y axes are horizontal and vertical respectively. Axes which are tangent (t) and normal (n) to the failure plane are oriented at an angle α with respect to the +x and +y axes. A positive value of α Is a counter-clockwise rotation, a negative value is a clockwise rotation.

Figure 2-4. Sign convention for geometry

(a) Assuming a potential failure surface which is based on the stratification, location and orientation, frequency and distribution of discontinuities of the foundation material, and the configuration of the substructure. The orientation of the failure surfaces for most wedges can be calculated directly by using the equations in paragraph 5-4 below.

(b) Dividing the assumed slide mass into a number of wedges, including a single structural wedge. See Figure 2-5 for the geometry of a typical wedge and the adjacent wedges associated with it.

(c) Drawing free body diagrams which show all the forces assumed to be acting on each wedge. The resultant forces acting on a typical wedge are shown in Figure 2-6. The free body diagram of a typical wedge is shown in Figure 2-7.

(d) Solving for the safety factor by either direct or iterative methods, and comparing it to the required safety factor.

(e) The analysis proceeds by assuming trial values of the safety factor and unknown inclinations of the slip path so the governing equilibrium conditions, failure criterion, and definition of safety factor are satisfied. An analytical or a graphical procedure may be used for this iterative solution.

(f) If it is only necessary to determine whether an adequate safety factor exists, this may be determined in a single step without the iterative process.

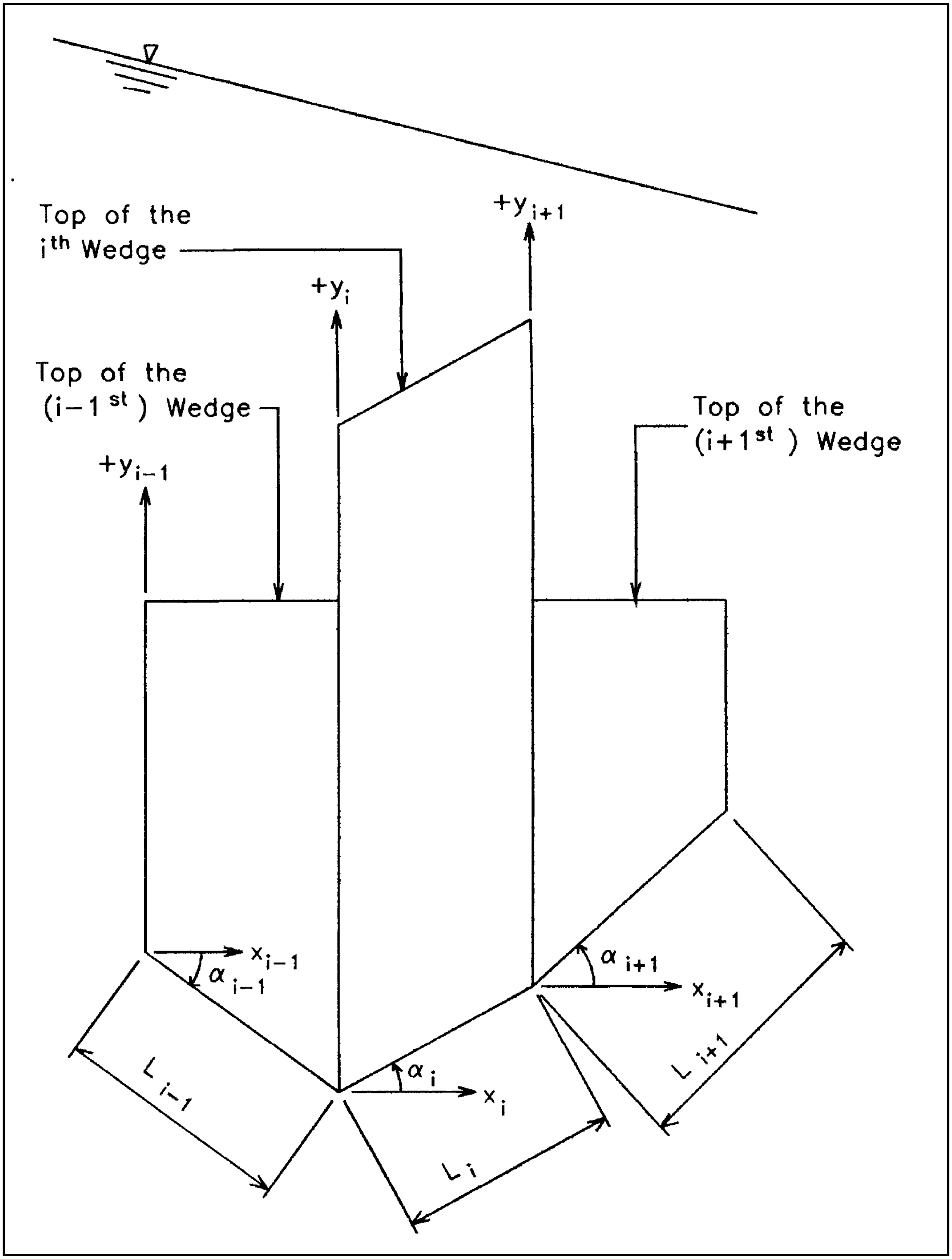


Figure 2-5. Geometry of the typical *ith* wedge and adjacent wedges

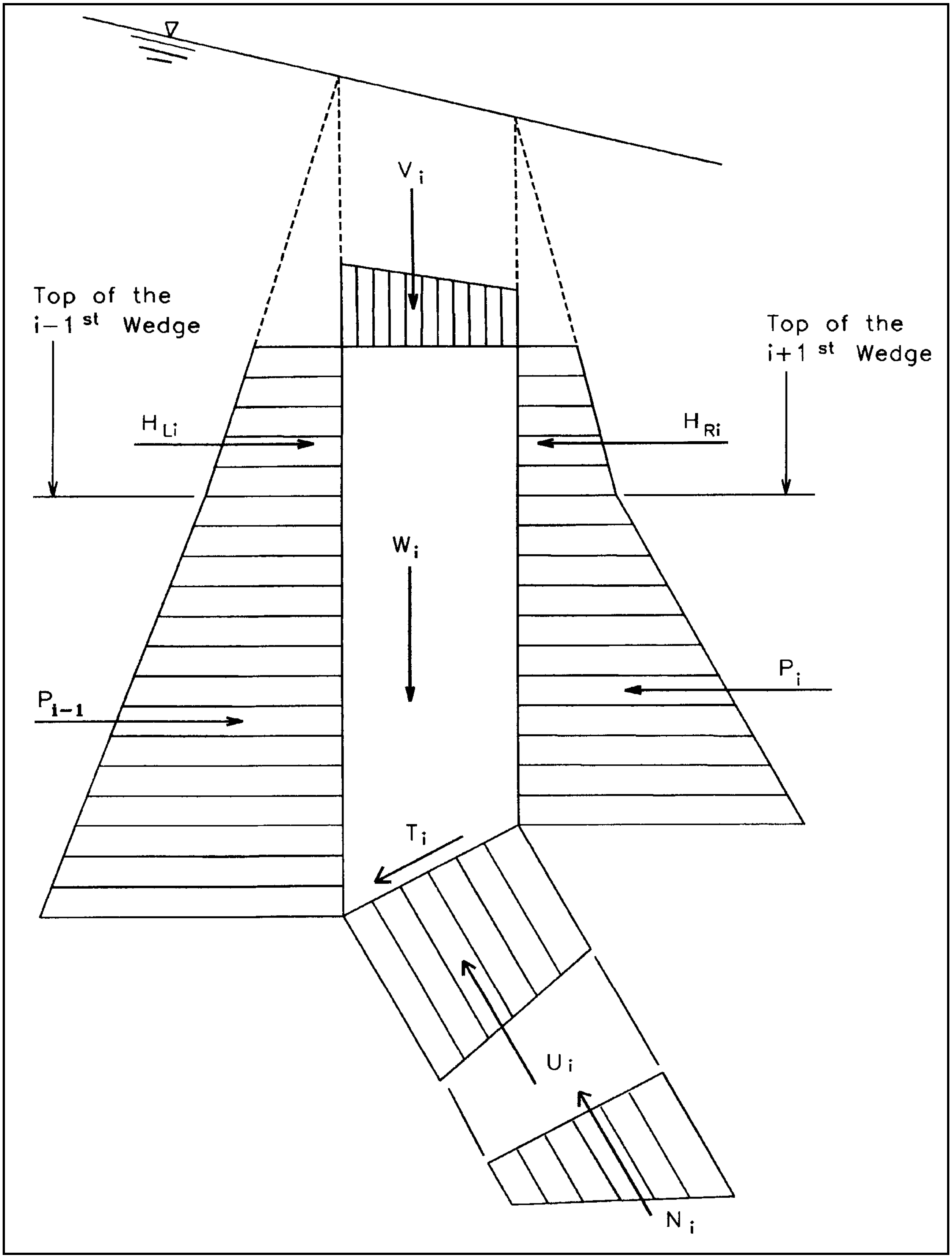


Figure 2-6. Distribution of pressures and resultant force acting on a typical wedge

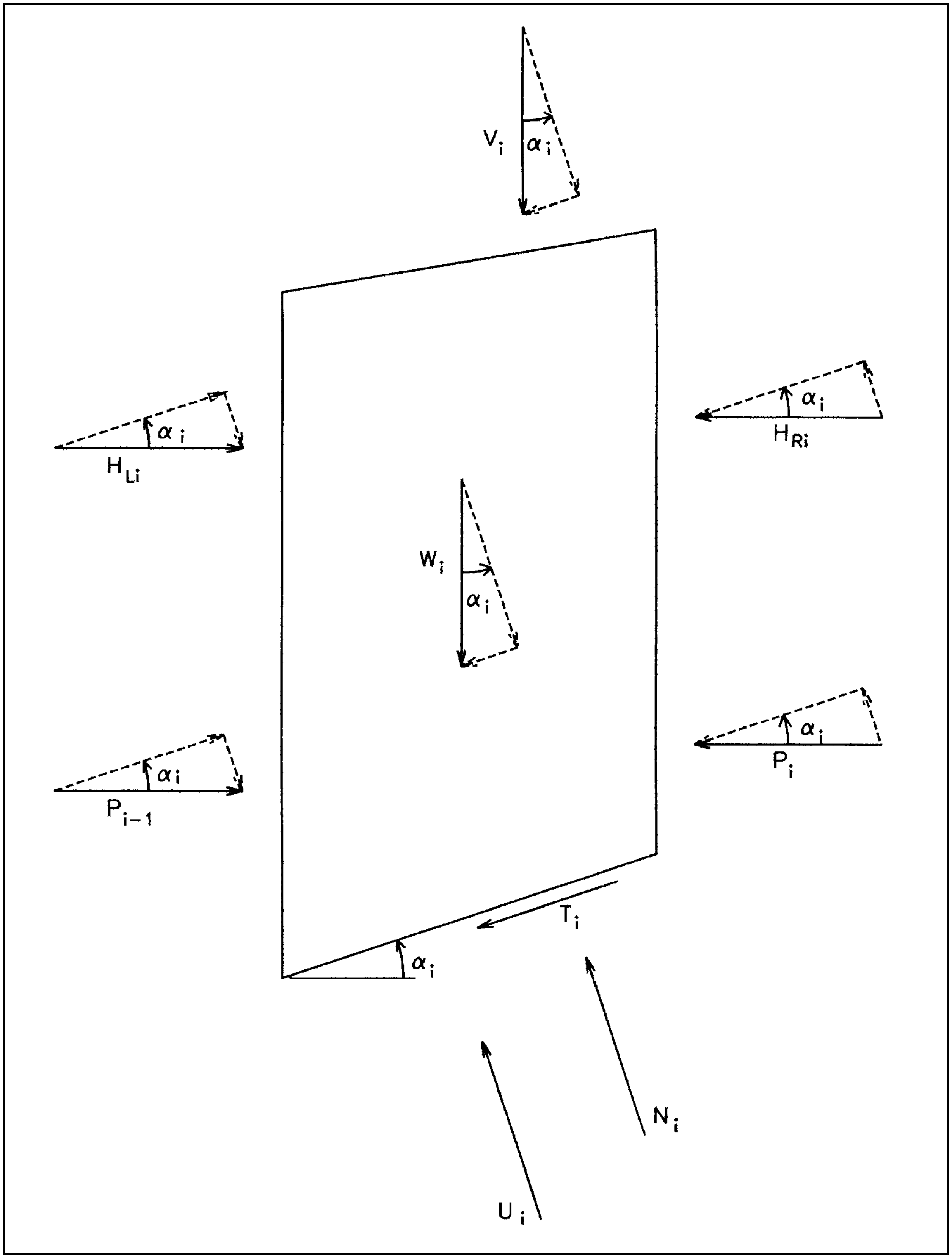


Figure 2-7. Free-body diagram of the *ith* wedge

d. Design considerations.

(1) Since all portions of the structure must slide as a unit, there can be only a single structural wedge. Discontinuities in the slip path beneath the structural wedge should be modeled by assuming an average slip plane along the base of the structural wedge.

(2) The interface between the group of driving wedges and the structural wedge is assumed to be a vertical plane located at the heel of the structural wedge and extending to the base of the structural wedge. The magnitudes of the driving forces depend on the actual values of the safety factor and the inclination angles (α) of the slip path. The inclination angles, corresponding to the maximum driving forces for each potential failure surface, can be determined by independently analyzing the group of driving wedges for a trial safety factor. In rock, the inclination may be predetermined by discontinuities in the foundation. The general equation only applies directly to driving wedges with driving forces that act parallel to the top surfaces of the wedges.

(3) The interface between the group of resisting wedges and the structural wedge is assumed to be a vertical plane located at the toe of the structural wedge and extending to the base of the structural wedge. The magnitudes of the resisting forces depend on the actual values of the safety factor and the inclination angles of the slip path. The inclination angles, corresponding to the minimum resisting forces for each potential failure mechanism, can be determined by independently analyzing the group of resisting wedges for a trial safety factor. When resisting force is used, special considerations may be required. Rock that may be subjected to high velocity water scouring should not be used unless amply protected. Also, the compressive strength of the rock layers must be sufficient to develop the wedge resistance. In some cases, wedge resistance should not be assumed without resorting to special treatment such as installing rock anchors.

(4) As stated previously, requirements for rotational equilibrium are not directly included in the general wedge equation. For some load cases, the normal component of the resultant applied loads will lie outside the kern of the base area, and a portion of the structural wedge will not be in contact with the foundation material. The sliding analysis should be modeled for these load cases to reflect the following secondary effects due to coupling of sliding and rotational behavior.

(a) The uplift pressure on the portion of the base which is not in contact with the foundation material should be a uniform value which is equal to the hydrostatic pressure at the adjacent face, (except for instantaneous load cases such as due to seismic forces).

(b) The cohesive component of the sliding resistance should only include the portion of the base area which is in contact with the foundation material.

e. Required factors of safety. The minimum required factors of safety shall be those determined by the methods presented in Chapter 3 of this EM. Any relaxation of these values will be accomplished only with the approval of CECW-ED and should be justified by comprehensive foundation studies of such nature as to reduce uncertainties to a minimum.

f. Derivation of governing wedge equation.

(1) Nomenclature for the terms used in the figures and following equations:

ϕ_i = internal friction angle of material in *ith* wedge.

c_i = cohesive strength of material in *ith* wedge.

W_i = weight of material in *ith* wedge. Based on moist unit weight above water table and buoyant unit weight below water table for earth wedges.

V_i = surcharge load acting on *ith* wedge. Should include the vertical components of lateral earth forces acting on the structural wedge.

H_{Li} = horizontal force on *ith* wedge, acting to the right. Includes the total horizontal water force on left side of structural wedge.

H_{Ri} = horizontal force on *ith* wedge, acting to left. Includes total horizontal water force on right side of structural wedge.

U_i = uplift. Water load acting normal to failure plane. Applied only to the structural wedge.

N_i = force acting normal to failure plane of *ith* wedge.

T_i = shear force acting parallel to failure plane of *ith* wedge.

P_i = horizontal force due to the *ith* wedge.

L_i = length of wedge base.

ß = top surface slope angle for an earth wedge; also wall friction angle. Does not apply to structural wedge.

(2) Equilibrium equations.

$$\Sigma F_n = 0$$

$$0 = N_i + U_i - W_i \cos\alpha_i - V_i \cos\alpha_i - H_{Li} \sin\alpha_i + H_{Ri} \sin\alpha_i - (P_{i-1} - P_i)(\sin\alpha_i - \tan\beta\cos\alpha_i) \tag{2-4}$$

$$N_i = (W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i + (P_{i-1} - P_i)(\sin\alpha_i - \tan\beta\cos\alpha_i)$$

$$\Sigma F_t = 0$$

$$0 = -T_i - W_i \sin\alpha_i - V_i \sin\alpha_i + (H_{Li} - H_{Ri})\sin\alpha_i + (P_{i-1} - P_i)(\cos\alpha_i + \tan\beta\cos\alpha_i) \tag{2-5}$$

$$T_i = (H_{li} - H_{Ri})\cos\alpha_i - (W_i + V_i)\sin\alpha_i + (P_{i-1} - P_i)(\cos\alpha_i + \tan\beta\sin\alpha_i)$$

(3) Mohr-Coulomb failure criterion.

$$T_F = N_i \tan\phi_i + c_i L_i \tag{2-6}$$

(4) Safety factor definition.

$$FS_i = \frac{T_F}{T_i} = \frac{N_i \tan\phi_i + c_i L_i}{T_i} \tag{2-7}$$

(5) Governing wedge equation.

$$FS_i = \frac{\left[(W_i + V_i)\cos\alpha_i - U_i + [(H_{Li} - H_{Ri})\sin\alpha_i + (P_{i-1} - P_i)(\sin\alpha_i - \tan\beta\cos\alpha_i)\right]\dfrac{\tan\phi_i}{FS_i} + c_i L_i}{[(H_{Li} - H_{Ri}) + (P_{i-1} + P_i)(\cos\alpha_i + \tan\beta\sin\alpha_i)} \tag{2-8}$$

$$(P_{i-1} - P_i) = \frac{\left[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i\right]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + C_i L_i}{\left(1 - \dfrac{\tan\beta\ \tan\phi_i}{FS_i}\right)\cos\alpha_i - \sin\alpha_i\left(\tan\beta + \dfrac{\tan\phi_i}{FS_i}\right)} \tag{2-9}$$

A negative value of the difference (P_{i-1} - P_i) indicates that the applied forces acting on the i^{th} wedge exceed the forces resisting sliding along the base of the wedge. A positive value of the difference (P_{i-1} - P_i) indicates that the applied forces acting on the i^{th} wedge are less than the forces resisting sliding along the base of that wedge.

g. Solution for the safety factor. The governing equation for (P_{i-1} - P_i) applies to the individual wedges. For the system of wedges to act as an integral failure mechanism, the safety factors of all wedges must be identical.

$$FS_1 = FS_2 = ...FS_{i-1} = FS_i = FS_{i+1} = ...FS_N$$

where

N = the number of wedges in the failure mechanism.

The actual safety factor (FS) for sliding equilibrium is determined by satisfying overall horizontal equilibrium (ΣF_H = 0) for the entire system of wedges.

$$\sum_{i=1}^{N} (P_{i-1} - P_i) = 0$$

and

$$P_0 \equiv 0 \qquad P_N \equiv 0$$

Usually, an iterative solution process is used to determine the actual safety factor for sliding equilibrium. An example of a typical static loading condition analysis for a multiple-wedge system is presented in Example D2 of Appendix D. Note that if $\Sigma F_H < 0$, the factor of safety is less than the trial factor of safety, and if $\Sigma F_H > 0$, the factor of safety is greater than the trial factor of safety.

2.11 Single-Wedge Sliding Analysis

Only the structural wedge is actively considered in the single-wedge sliding analysis. This is a simpler method which will usually produce the same results as the multiple-wedge method. The basic concepts are similar for both methods, but all driving and resisting wedges are replaced with earth and groundwater forces calculated directly, using the methods in Chapter 5. The single-wedge method is illustrated in Appendix D, example D1. These methods produce reasonably conservative estimates of the earth forces used for the sliding analysis and for other stability analyses and for structural design.